

# A VMD Based, Nonet and SU(3) Symmetry Broken Model For Radiative Decays of Light Mesons

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by

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in name of

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## Abstract

We present a VMD based model aiming to describe all radiative decays of light mesons. We show that the SU(3) breaking mechanism proposed by Bando, Kugo and Yamawaki (BKY), supplemented by nonet symmetry breaking in the pseudoscalar sector are sufficient to provide a nice description of all data, except the  $K^{*\pm}$  radiative width. It is also shown that nonet symmetry breaking has effects which cannot be disentangled from those produced by coupling of glue to the  $\eta'$  meson. Coupling of glue to  $\eta$  is not found to be required by the data. Assuming the  $K^{*\pm}$  radiative width is indeed at its presently accepted value necessitates to supplement the BKY breaking in a way which finally preserves an equivalence statement between the VMD approach to radiative decays and the Wess–Zumino–Witten Lagrangian.

# 1 Introduction.

It is a long standing problem to define a framework in which all radiative decays of light flavor mesons can be accurately accounted for. A few kinds of different models have been proposed so far. The most popular modelling is in terms of magnetic moments of quarks [1, 2]. Another traditional approach is to use SU(3) relations among coupling constants [3]. This yields reasonable descriptions of radiative decays [4], though the success is not complete.

The O'Donnell model assumes exact SU(3) flavor symmetry, while nonet (or U(3) flavor) symmetry is explicitly broken. As it follows from a quite general conceptual framework, this model is widely independent of detailed dynamical properties and assumptions. This model covers all couplings like  $PV\gamma$  but lacks to describe  $P\gamma\gamma$  decays which remain unrelated.

Recently, several models of other kinds have been proposed [5, 6], motivated by effective Lagrangian approaches to the interactions of vector mesons [7, 8], with various kinds of SU(3) breaking schemes [5, 6, 9, 10]. However, breaking of nonet symmetry in phenomenological models has got little attention [3, 4].

The study of radiative decays of light flavor mesons is also connected with the long standing problem of  $\eta/\eta'$  mixing [4, 11, 12] and to its possible association with a glue component inside light mesons [6, 13]. Recent developments advocate a more complicated  $\eta/\eta'$  mixing scheme [14, 15].

As effects of SU(3) symmetry breaking are clearly observed in the data on radiative decays of light mesons [4, 6], they have surely to be accounted for. We do it following the BKY breaking mechanism [9, 10]. In this way, by means of the FKTUY Lagrangian and of the BKY breaking scheme, we can construct a Lagrangian formulation of the O'Donnell model and extend it to the case where the SU(3) flavor symmetry is broken. This, additionally, provides an algebraic connection between  $VP\gamma$  and  $P\gamma\gamma$  coupling constants.

What is presented here is an account of a work [17] mainly devoted to a study of radiative of light flavor mesons within the general VMD framework of the hidden local symmetry model [7] (hereafter referred to as HLS) and its anomalous sector [8] (hereafter referred to as FKTUY). All details and previous references can be found there. The perspective of this talk is however somewhat different, due to recent developments presented in Ref. [18].

The HLS model is an expression of the Vector Meson Dominance (VMD) assumption ; it thus gives a way to relate the radiative decay modes  $VP\gamma$  to each other and to the  $P\gamma\gamma$  decays for light mesons, by giving a precise meaning to the equations shown in Fig. 1.

One can try to estimate naively this relation using other information collected in the Review of Particle Properties [19].

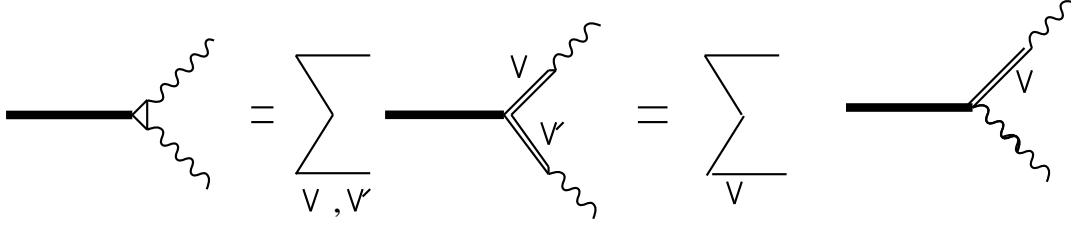


Figure 1: Graphical representation of the relation among various kind of coupling constants.  $V$  and  $V'$  stand for the lowest lying vector mesons ( $\rho^0$ ,  $\omega$ ,  $\phi$ ) ; the internal vector meson lines are propagators at  $s = 0$  and are approximated by the corresponding tabulated [19] masses squared.

Mode	VMD prediction	PDG
$\pi^0 \rightarrow \gamma\gamma[\text{eV}]$	$10.73 \pm 1.20$	$7.74 \pm 0.50$
$\eta \rightarrow \gamma\gamma[\text{keV}]$	$0.62 \pm 0.18$	$0.46 \pm 0.04$
$\eta' \rightarrow \gamma\gamma[\text{keV}]$	$5.10 \pm 0.76$	$4.27 \pm 0.19$

**Table 1** : Partial decay widths of the pseudoscalar mesons, as reconstructed from VMD, using the  $VP\gamma$  measured couplings, and their direct accepted measurements [19].

The results given in Table 1 are quite impressive ; this is indeed not a fit, but mere algebra. Thus, all systematics can pill up and, moreover, the meson masses used in order to estimate the propagators at  $s = 0$  are simply the (Breit–Wigner) accepted masses [19] ; this is surely a very crude assumption, at least for the  $\rho^0$  meson. However, this exercise teaches us that the central hint of the Vector Meson Dominance assumption is sharply grounded and this motivates to try going beyond as much as possible.

A further comment is of relevance concerning the VMD prediction for the  $\pi^0$  decay width. Actually, the two–photon width of the  $\pi^0$  can be computed, as sketched in Fig. 1, from two different ways since the basic  $VVP$  diagram is  $\pi^0\rho^0\omega^I$ , where  $\omega^I$  is the ideal (purely non–strange) combination of the (physical)  $\omega$  and  $\phi$  fields. A first estimate is thus obtained from the coupling  $\pi^0\rho^0\gamma$  (here the hidden vector meson line is surely  $\omega^I$ ) and is  $12.79 \pm 2.59$  eV ; a second estimate is obtained from using instead the couplings  $\pi^0\omega\gamma$  and  $\pi^0\phi\gamma$  (here the hidden vector meson line is surely  $\rho^0$  for both) and is  $8.86 \pm 0.29$  eV. The qualitative difference of both estimates reflects problems with the  $\rho$  mass definition which will not be examined here. What is given in Table 1 is simply the mean value of both estimates.

## 2 An Exact SU(3) Symmetry Framework

The formalism which describes the decays  $V \rightarrow P \gamma$  and  $P \rightarrow V \gamma$  within an exact SU(3) flavor symmetry framework has been given by P. O'Donnell in Ref. [3]. The corresponding decay amplitudes can be quite generally written as

$$T = g_{VP\gamma} \epsilon_{\mu\nu\rho\sigma} k^\mu q^\nu \varepsilon^\rho(V) \varepsilon^\sigma(\gamma) \quad (1)$$

using obvious notations. This expression can be found by relying entirely on gauge invariance and does not require the help of any specific Lagrangian.

Using SU(3) symmetry, the coupling strengths  $g_{VP\gamma}$  between physical vector and pseudoscalar mesons in radiative decays are expressed in terms of two angles ( $\theta_V$  and  $\theta_P$ ) which describe the mixtures of singlet and octet components, and of three coupling constants ( $g_{V_8 P_8 \gamma}$ ,  $g_{V_0 P_8 \gamma}$  and  $g_{V_8 P_0 \gamma}$ ); as the photon behaves like an SU(3) octet, this cancels out the possible coupling  $g_{V_0 P_0 \gamma}$ . We do not reproduce here the expressions for the  $g_{VP\gamma}$  in terms of the elementary couplings  $g_{V_i P_j \gamma}$  and the mixing angles; they can be found in Ref. [3] and in Appendix A7 of Ref. [4], where a misprint has been corrected.

One generally uses the representation of the matrices for vector mesons ( $V$ ) and pseudoscalar mesons ( $P$ ) in the  $\{u, d, s\}$  basis. Their expressions are quite classical and can be found, for instance, in Refs. [10, 17]. It is usual to express the relevant matrix elements of  $V$  in terms of ideally mixed states ( $\omega^I, \phi^I$ ), while it is as traditional to express the isoscalar mesons fields in  $P$  in terms of the conventional octet and singlet components ( $\pi_8, \eta_0$ ). These are not the physical states ( $\omega, \phi, \eta, \eta'$ ) which are generated from ideally mixed states by means of rotations. The rotation angles from singlet and octet states to the physically observed mesons are traditionally named  $\theta_V$  and  $\theta_P$ . These well known relations can be found in Refs. [3, 10, 17, 19].

With these definitions for the field matrices, the effective FKTUY Lagrangian which describes the anomalous sector of the HLS model is [8]

$$\mathcal{L} = -\frac{3g^2}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} \text{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma P]. \quad (2)$$

The universal vector meson coupling  $g$  is tightly related to the coupling of the  $\rho$  meson to a pion pair and  $f_\pi = 92.41$  MeV is the usual pion decay constant. The partial widths for all  $VP\gamma$  and  $P\gamma\gamma$  modes are derived herefrom, using also the  $V\gamma$  transition amplitudes and the expressions for the vector meson masses given in the standard (non-anomalous) HLS Lagrangian [7, 17]. Actually, the coefficient in Rel. (2) is fixed [8] in order that this Lagrangian leads to the usual expression for the amplitude of  $\pi^0 \rightarrow \gamma\gamma$ .

At this point, it should be emphasized that exact SU(3) symmetry is not in conflict with releasing the condition of nonet symmetry (which corresponds to the stronger U(3) symmetry) usually stated in effective Lagrangian models for both the vector and pseudoscalar meson sectors [7, 8]. The field matrices  $V$  and  $P$  can be written  $V = V_8 + V_0$  and  $P = P_8 + P_0$ , in order to exhibit their (matrix) octet and singlet mixtures. It can be checked, that the O'Donnell model [3, 17] can be generated by simply replacing in Rel. (2) the (nonet symmetric) vector and pseudoscalar field matrices by

$$\begin{cases} P = P_8 + xP_0 \\ V = V_8 + yV_0 \end{cases} \quad (3)$$

Referring to [3,17], the basic coupling constants of the model are

$$\begin{cases} g_{V_8 P_8 \gamma} = G = -\frac{3eg}{8\pi^2 f_\pi} \\ g_{V_0 P_8 \gamma} = yG \\ g_{V_8 P_0 \gamma} = xG \end{cases} \quad (4)$$

So, one can choose  $x$ ,  $y$  and  $G$  in addition to the mixing angles as free parameters to be determined from fit to data. This identification is already interesting, as it relates the main coupling constant  $G$  in the O'Donnell model to more usual quantities ( $g$ ,  $f_\pi$ ). Finally,  $x$  and  $y$  are the deviations from nonet symmetry in respectively the pseudoscalar and vector sectors.

It was phenomenologically checked<sup>1</sup> [4], that radiative decays of light mesons are consistent with nonet symmetry in the vector sector ; thus, data accommodate  $y = 1$  quite naturally. However, the same analysis concluded to a small but significant departure (slightly more than  $4\sigma$ ) from nonet symmetry in the pseudoscalar sector, which corresponds to  $x \simeq 0.90$ . From a physics point of view, the resulting picture was close to be acceptable, as only two decay modes,  $K^{*0} \rightarrow K^0 \gamma$  and  $\phi \rightarrow \eta \gamma$ , were not satisfactorily accounted for (see in Ref. [4] the “internal fit” entry of Table 8). Qualitatively, the former disagreement could be due to SU(2) symmetry breaking because of the  $K^{*0}$  quark content<sup>2</sup>. However, the disagreement about the later mode (more than a factor of 2) is clearly a signal of unaccounted for SU(3) breaking effects, since the branching fraction for  $\phi \rightarrow \eta \gamma$  has been recently confirmed twice [19].

Therefore the O'Donnell model [3,4] is already close enough to observations that one may conclude that nonet symmetry breaking is a working concept and that only some amount of SU(3) breaking is needed in order to achieve a quite consistent description of radiative decays. This is the main purpose of the work presented here. It should be re-emphasized, however, that the original model of O'Donnell did not relate  $VP\gamma$  and  $P\gamma\gamma$  modes, while the connection just sketched of this model with the HLS approach provides the lacking algebraic connection.

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<sup>1</sup>The quoted deviations of  $x$  and  $y$  from unity have been confirmed by the present analysis. For instance, releasing  $y$  in the present U(3), SU(3) broken model leads to  $y = 0.996 \pm 0.033$ , quite consistent indeed with  $y = 1$  and thus with nonet symmetry.

<sup>2</sup>However, a factor of 1.5 at the rate level, *i.e.* a factor 1.25 at the level of coupling constants, could look somehow beyond expectable SU(2) breaking effects.

### 3 SU(3) Breaking of the HLS–FKTUY Model

The SU(3) symmetry breaking (referred to as BKY) we use originates from Refs. [7,9]. Briefs accounts and some new developments can be found in Refs. [5,10], connected more precisely with the anomalous sector [8].

#### 3.1 SU(3) Breaking Mechanism of the HLS Model

Basically, the SU(3) breaking scheme we use has been introduced by Bando, Kugo and Yamawaki [9] (referred to as BKY) and has given rise to a few variants [5,10]. In order to recover the charge normalization  $F_{K^+}(0) = 1$ , even after breaking of SU(3) flavor symmetry, one is obliged to define a renormalized pseudoscalar field matrix  $P'$  in terms of the bare one  $P$  by

$$P' = X_A^{1/2} P X_A^{1/2}, \quad (5)$$

where the breaking matrix  $X_A$  writes  $\text{diag}(1, 1, 1 + c_A)$  and we have [9,10]

$$\ell_A \equiv 1 + c_A = \left( \frac{f_K}{f_\pi} \right)^2 = 1.495 \pm 0.030 \quad , \quad (6)$$

The numerical value just given is deduced from the experimental information quoted in Ref. [19].

#### 3.2 A Phenomenological Lagrangian for Radiative Decays

Following FKTUY [8], the anomalous U(3) symmetric Lagrangian describing  $PVV$  interactions and, using the non-anomalous Lagrangian [10,17],  $PV\gamma$  and  $P\gamma\gamma$  transitions is given by Eq. (2). Its full expansion can be found in Ref. [10]. Postulating that the same formulae apply when breaking nonet symmetry ( $x \neq 1$ ), is confirmed by its formal agreement with the O'Donnell derivation of the coupling constant formulae.

However, breaking the SU(3) symmetry à la BKY, also implies that we have to re-express the FKTUY Lagrangian in terms of the renormalized matrix  $P'$ , instead of the bare one  $P$ ; this is done using Rel. (5), with the (fixed) parameter given in Rel. (6). We remind that breaking nonet symmetry means that  $P'$  is also modified by the replacement  $\eta_0 \longrightarrow x\eta_0$ .

Propagating this field renormalization down to the FKTUY Lagrangian writes

$$\mathcal{L} = -\frac{3g^2}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} \text{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma X_A^{-1/2} P' X_A^{-1/2}]. \quad (7)$$

Then, the VVP Lagrangian is changed in a definite way by the symmetry breaking parameter  $\ell_A$  defined above (see Eq. (6)) and supposed to have a well understood numerical value (practically 1.5).

The expanded form of this Lagrangian is given in the Appendix of Ref. [17]. In principle, from this Lagrangian, one is able to construct the decay amplitudes for the  $V \rightarrow P\gamma$ ,  $P \rightarrow V\gamma$ ,  $V \rightarrow e^+e^-$  and  $P \rightarrow \gamma\gamma$  processes. They can be found in the appendix in Ref. [17].

### 3.3 The VMD Description of $\eta/\eta' \rightarrow \gamma\gamma$ Decays

The expression for the decay amplitude  $G_{\eta\gamma\gamma}$  are given by Eq (42) in Ref. [17]. It compares well with the corresponding expression of Ref. [22] deduced from the Nambu–Jona–Lasinio model. This shows that breaking parameters in this reference, originally expressed as functions of effective quark masses, also get an expression in terms of  $f_\pi/f_K$ . More precisely, as remarked in Ref. [22], in the (chiral) limit of vanishing meson masses, their breaking parameter, which can be formally identified to our  $Z = [f_\pi/f_K]^2$ , is simply the ratio  $m_q/m_s$  ( $q$  stands for either of  $u$  or  $d$  which have equal masses if SU(2) flavor symmetry is fulfilled) of the relevant effective masses of quarks.

With this respect, a surprising connection could be made with the traditional description of radiative decays using quark magnetic moments [1]. Indeed, the present fit values for these are [2] :

$$\mu_u = 1.852 \quad \mu_d = -0.972 \quad \mu_s = -0.630 \quad (8)$$

in units of Bohr magnetons. These magnetic moments corresponds to the following quark (effective) masses

$$m_u = 355.1 \text{ MeV} \quad m_d = 337.4 \text{ MeV} \quad m_s = 522.8 \text{ MeV} \quad (9)$$

It is indeed a point that  $m_s/m_u = 1.47$ ,  $m_s/m_d = 1.55$  compare well with  $[f_K/f_\pi]^2 = 1.495$ , as it can be guessed from the remark by Takizawa *et al.* [22]. Whether, this is accidental, or reveals a deeper property is an open question.

### 3.4 The WZW Description of $\eta/\eta' \rightarrow \gamma\gamma$ Decays

More interesting is that, starting from broken HLS and FKTUY, we recover the traditional form for these amplitudes, (*i.e.* the one mixing angle expressions of Current Algebra [11,12,23]). Using these standard expressions, one indeed gets through identification :

$$\frac{f_\pi}{f_8} = \frac{5 - 2Z}{3} \quad , \quad \frac{f_\pi}{f_0} = \frac{5 + Z}{6}x \quad , \quad (10)$$

where  $Z = [f_\pi/f_K]^2$ . This shows that, in the limit of SU(3) symmetry, we have  $f_8 = f_\pi$  and  $f_0 = f_\pi/x$ , and that  $f_0 = f_8 = f_\pi$  supposes that there is no symmetry breaking at all.

Actually, these formulae mean that, instead of going through the whole machinery of VMD by starting from the broken FKTUY Lagrangian, one could get these coupling constants for two-photon decays of the pseudoscalar mesons by starting from the WZW Lagrangian [24,25]. Indeed, this can be written

$$\mathcal{L}_{WZW} = -\frac{e^2}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho A_\sigma \text{Tr}[Q^2 P] \quad (11)$$

where  $Q = \text{diag}(2/3, -1/3, -1/3)$  is the quark charge matrix,  $A$  the electromagnetic field and  $P$  is the bare pseudoscalar field matrix. Changing to  $P'$  through Rel. (5), allows indeed to recover directly (and trivially) the couplings given in the Appendix.

This illustrates clearly that, what is named  $f_8$  in the Current Algebra [23] expressions for  $\eta/\eta'$  decays to two photons, can be expressed solely in terms of  $f_\pi$  and  $f_K$ , in a way which fixes its value to  $f_8 = 0.82f_\pi$ . The fact that the WZW Lagrangian leads to the same results as the FKTUY Lagrangian simply states their expected equivalence when deriving two-photon decays amplitudes for pseudoscalar mesons.

On the other hand, the SU(3) sector of Chiral Perturbation Theory (ChPT) predicts [11, 26]  $f_8/f_\pi \simeq 1.25$ . One could thus think to a contradiction [17] between VMD (or FKTUY) and WZW on the one hand, and ChPT on the other hand. However, it happens [18] that this is a misleading appearance due to an inconsistency between defining the decay constants and mixing angle in agreement with Current Algebra – definitions recovered by VMD and WZW as illustrated above – and current ChPT definitions.

## 4 Fitting Decays Modes with the Broken Model

In this section, we focus on the model for coupling constants given by Eqs. (39) to (45) of Ref [17], and use them for a fit to radiative decays of light mesons. the corresponding data are all taken<sup>3</sup> from the Review of Particle Properties [19].

From what is reported several times in the literature [2, 4, 6], one might expect potential problems with one or both  $K^*$  decay modes. Therefore, we have followed the strategy of performing fits of all radiative decay modes except for these two. In all fits performed with the model described above, we have found that the prediction for  $K^{*0} \rightarrow K^0\gamma$  is in fairly good agreement with the corresponding measurement [19], while the expected value for  $K^{*+} \rightarrow K^+\gamma$  is always at about  $5\sigma$  from the accepted value [19]. Therefore, in the fits referred to hereafter, the process  $K^{*+} \rightarrow K^+\gamma$  has been removed.

The difficulty met with this decay mode in several studies mentioned above could cast some doubt on the reliability of this measurement, performed using the Primakoff effect. However, it cannot be excluded that this measurement is indeed correct and that the reported disagreement simply points toward the need of refining the models. This will be discussed in Section 7.

### 4.1 SU(3) Breaking and the Value of $f_K$

The key parameter associated specifically with the breaking of SU(3) flavor symmetry is the BKY parameter  $\ell_A$ , expected to be equal to  $[f_K/f_\pi]^2$  (see Rel. (6)) . As starting point in our fit, we have left free all parameters :  $G$ ,  $x$ ,  $\theta_V$ ,  $\theta_P$  and  $\ell_A$ . We thus got a nice fit probability ( $\chi^2/\text{dof} = 10.74/9$ ) and the result we like to mention from this fit is

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<sup>3</sup>We have however used in the fits [17], as partial width for  $\eta \rightarrow \gamma\gamma$ , the mean value of the measurements reported by  $\gamma\gamma$  experiments, instead of the PDG [19] value, which is affected by the single existing Primakoff measurement.



$$\frac{f_K}{f_\pi} = 1.217^{+0.021}_{-0.019}. \quad (12)$$

which is almost exactly the value expected from the known ratio  $f_K/f_\pi$ . This gives, of course, a strong support to the BKY breaking mechanism [9, 10].

This result strongly suggests that one can reasonably fix  $\ell_A = 1.50$  (at its physical value). Then, the single free breaking parameter which influences the coupling constants in radiative decays, beside mixing angles, is the nonet symmetry breaking parameter  $x$ .

## 4.2 The Fit Parameter Values

Except for the two mixing angles, we only have two free parameters to fit the data set, as in the unbroken case [4]. One,  $G$ , is connected with the vector meson universal coupling  $g$ , the other is the nonet symmetry breaking parameter  $x$ . The final fit exhibits a very good quality ( $\chi^2/dof = 10.9/10$ ), corresponding to a 44% probability, and the best values and errors for the main parameters ( $\ell_A$  is fixed to 1.5) are

$$\left\{ \begin{array}{ll} G &= 0.704 \pm 0.002 \quad [\text{GeV}]^{-1} \\ x &= 0.917 \pm 0.017 \\ \theta_V &= 31.92 \pm 0.17 \quad [\text{deg.}] \\ \theta_P &= -11.59 \pm 0.76 \quad [\text{deg.}] \end{array} \right. \quad (13)$$

The nonet symmetry breaking parameter is  $x = 0.92 \pm 0.02$ , confirming a previous analysis [4]. The value for  $G$  is also in nice agreement with the previous analysis of Ref. [4]. The vector mixing angle is found at 3.4 degrees below its ideal value<sup>4</sup> and agrees with predictions [27]. The mixing angle of pseudoscalar mesons coming out from fit points toward a small deviation from the Gell–Mann–Okubo mass relation [11, 18].

## 4.3 The One Angle $\eta/\eta'$ Mixing Scheme from VMD

As discussed above the model we propose, which relies on the VMD approach of Refs. [7, 8] with fixed SU(3) breaking à la BKY [9, 10], leads to (one angle) formulae for the  $\eta/\eta' \rightarrow \gamma\gamma$  decay amplitudes. These can be identified with the corresponding Current Algebra standard expressions and we have recalled that they can also be directly derived from the WZW Lagrangian. This justifies the identification shown in Eq. (10) for the singlet and octet coupling constants. One should note that nonet symmetry breaking does not modify the formulae substantially. In this case, we obtain together

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<sup>4</sup> Let us, however, remind that this value relative to ideal mixing is the consequence of our choice  $\phi^I = -|s\bar{s}\rangle$ .

with  $\theta_P = -11.59^\circ \pm 0.76^\circ$  :

$$\frac{f_8}{f_\pi} = 0.82 \pm 0.02 \quad , \quad \frac{f_1}{f_\pi} = 1.15 \pm 0.02 \quad (14)$$

using Eq. (12), and the fit result for  $x$ .

Actually, it happens that a low value for  $f_8/f_\pi$  and a low absolute value for the pseudoscalar mixing angle  $\theta_P$  are correlated properties [6, 17].

One can also perform a fit of the  $VP\gamma$  processes in isolation in order to get estimates for  $x$  and  $\theta_P$ , free of any influence of the  $P\gamma\gamma$  processes. This allows to check the conceptual relation between  $VP\gamma$  and  $P\gamma\gamma$  inferred from VMD. Using the formulae of Ref. [17], one can indeed reconstruct the VMD expectations for the  $P\gamma\gamma$  modes. The interesting point here, compared with what is shown in Table 1, is that the fit procedure improves the parameter values associated with the  $VP\gamma$  modes. The results are shown in Table 2.

Then, the HLS approach we have developed, even restricted to the  $VP\gamma$  processes is indeed able to predict quite nicely the  $P\gamma\gamma$  partial widths. The effects of fitting can easily be understood by comparing the corresponding information and accuracies in Tables 1 and 2. Thus, the VMD formulae (which are also those obtained [10] by breaking à la BKY the Wess-Zumino Lagrangian) provide the traditional one-angle scheme of the former Current Algebra. No need for a more complicated mixing pattern [14, 15] emerges from the data on radiative decays.

Mode	VMD Fit	PDG	Comment
$\eta \rightarrow \gamma\gamma$ [keV]	$0.464 \pm 0.026$	$0.514 \pm 0.026$	$\gamma\gamma$
		$0.46 \pm 0.04$	PDG mean
		$0.324 \pm 0.046$	Primakoff
$\eta' \rightarrow \gamma\gamma$ [keV]	$4.407 \pm 0.233$	$4.27 \pm 0.19$	PDG mean

**Table 2** : Partial decay widths of the  $\eta/\eta'$  mesons, as reconstructed from fit to solely the radiative decays  $VP\gamma$  (leftmost data column) and their direct measurements [19] (rightmost data column).

## 5 Is There a Glue Component Coupled to $\eta/\eta'$ ?

As stated in the Introduction, the precise content of the pseudoscalar singlet component in  $\eta/\eta'$  mesons is somehow controversial. One cannot indeed exclude the interplay of the usual singlet  $\eta_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$  with other SU(3) singlet states [6, 13], which could be glueballs or some  $c\bar{c}$  admixture, or both. Let us assume the existence of such an

additional singlet state that will be denoted  $gg$ , in order to make formally the connection with its possibly being a gluonium. Then we should allow for mixing of these two possible singlet component with the standard SU(3) octet  $\pi_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$

## 5.1 The $\eta/\eta'$ Mesons in Terms of Octet and Singlet States

An appropriate parametrization for the mixing of  $(\pi_8, \eta_0, gg)$  into physical pseudoscalar meson states denoted  $(\eta, \eta', \eta'')$  is needed. The symbol  $\eta''$  for the third partner of the doublet  $(\eta, \eta')$  simply means that we consider premature to try identifying it. This is done by means of an orthogonal matrix transform ; an appropriate parametrization of this matrix is the Cabibbo–Kobayashi–Maskawa matrix (with no complex phase). This transform is given by Eq (2) in Ref. cited. It depends on 2 angles, in addition to the usual  $\theta_P$ , which have been named  $\beta$  and  $\gamma$ . They are such that the vanishing of  $\beta$  and  $\gamma$  gives smoothly the usual mixing pattern of the  $(\eta, \eta')$  doublet (with one angle  $\theta_P$ ) and the decoupling of the additional singlet. Setting  $\beta = 0$  cancels out glue inside  $\eta$  only, while  $\gamma = 0$  removes any glue inside the  $\eta'$  only. Then this transform allows for analyzing the interplay of an additional singlet (named here glue) in a continuous way for both the  $\eta$  and the  $\eta'$  mesons.

## 5.2 Nonet Symmetry Breaking versus Glue

Up to now, we have illustrated that the BKY breaking was a fundamental tool in order to describe all data concerning radiative and two-photon decays of light mesons. The other central result of our fitting model concerns the unavoidable need of about 10% breaking of nonet symmetry in the pseudoscalar sector ( $x \simeq 0.9$ ). This could well be a fundamental property.

However, the observed nonet symmetry breaking could also be an artefact of the model above, reflecting physical effects intrinsically ignored. In this Section we examine the interplay of nonet symmetry breaking and a possible glue component. The  $VP\gamma$  and  $P\gamma\gamma$  coupling constants have been determined (see Eqs (46) and (47) in Ref. [17]).

A phenomenological study of these relations, which includes SU(3) breaking, nonet symmetry breaking and glue has been performed with the following conclusions : **1** The BKY breaking is still found determined by the value of  $f_K/f_\pi$  ; it can thus be fixed as previously done. **2** Nonet symmetry breaking and glue are intimately connected and reveal a correlation close to the 100% level. This second remark does not mean that nonet symmetry breaking and glue (or any additional singlet) are physically equivalent. The single appropriate conclusion is rather that, in order to conclude firmly about each of these twin phenomena, one needs relatively precise information on the other [18].

However, a few additional remarks can be drawn [17]. One can analyze how coupling to glue evolves as a function of a fixed nonet symmetry breaking level. This is shown in Fig 2. One clearly sees there that no need for glue is exhibited by the data if  $x \simeq 0.9$  ( $\beta$  and  $\gamma$  can be chosen equal zero without hurting the data). At  $x \simeq 1$  and somewhat

above, the angle  $\beta$  is still consistent with zero, pointing to the fact that one can hardly claim the need for glue in the  $\eta$  meson. However, somewhat above  $x \simeq 0.9$ , the level of glue in  $\eta'$  is a rising function of  $x$ , as shown by the steep dependence of  $\gamma$  upon  $x$ . In order to fix one's idea, if for some reason  $x \simeq 1$  has to be preferred, then one can express the glue fraction in  $\eta'$  by  $\cos^2 \gamma \simeq 0.20$  (at  $x = 1$ ).

In view of all this, beside the model with no glue and with a small breaking of nonet symmetry, we have studied the case of glue in only the  $\eta'$  meson (setting  $\beta = 0$ ) and no breaking of nonet symmetry. Moreover, the above remarks justify to perform a fit by fixing (as before) the SU(3) breaking at its expected value ( $\ell_A = 1.5$ ), choosing also  $x = 1$  and  $\beta = 0$  (in order to lessen at most correlation effects). In this case we have exactly the same number of parameters as in the previous set of fits. The corresponding fit results show a nice quality ( $\chi^2/dof = 10.5/10$ ), quite equivalent to the no-glue case. The predicted branching fractions are discussed in Section 6 below.

As major conclusions of this section, one can first assert that a possible glue content inside the  $\eta$  is not requested by the data. A significant glue content inside the  $\eta'$  is possible, however subject to the actual level of nonet symmetry breaking [18].

## 6 Estimates for Branching Fractions from Fits

We give and discuss here the reconstruction properties of the the two variants of our model, both discussed above. These are **i**/ nonet symmetry breaking supplemented by a fixed SU(3) breaking (BKY) and **ii**/ a fixed SU(3) breaking (BKY) with glue inside the  $\eta'$  replacing nonet symmetry breaking.

We now compare the branching fractions predicted by these two solutions to the accepted branching fractions as given in the Review of Particle Properties [19]. They are computed according to the formulae for coupling constants given in Ref. [17]. In Table 3, we list the information for radiative decays. The first remark which comes to mind by comparing the two model reconstructions is that their predictions are close together (see the first two data column). This illustrates clearly the numerical equivalence of coupling to glue and nonet symmetry breaking.

The relative disagreement of  $\eta' \rightarrow \rho^0 \gamma$  with accepted values [19] is actually an interesting artefact. Indeed, what has been submitted to fit is not the branching fraction given in Ref. [19], but the corresponding coupling constant extracted by the Crystal Barrel Collaboration in [28]. The reason for this is that the (published) branching fraction for  $\eta' \rightarrow \rho^0 \gamma$  is influenced by a non-resonant contribution originating from the box anomaly [4, 23–25] for the vertex  $\eta' \pi^+ \pi^- \gamma$ . This is not accounted for in the VMD model of [8] and has thus to be removed. Actually this process contributes to the total  $\chi^2$  by only  $\simeq 0.5$ . Moreover, its importance is not that decisive that it influences the fit results dramatically. This last information, has been tested by removing the  $\eta' \rightarrow \rho^0 \gamma$  decay from fit data.

On the other hand, even if quite acceptable, the reconstruction for  $\eta \rightarrow \gamma \gamma$  branching fraction is influenced by having used for this decay mode the mean value of the mea-

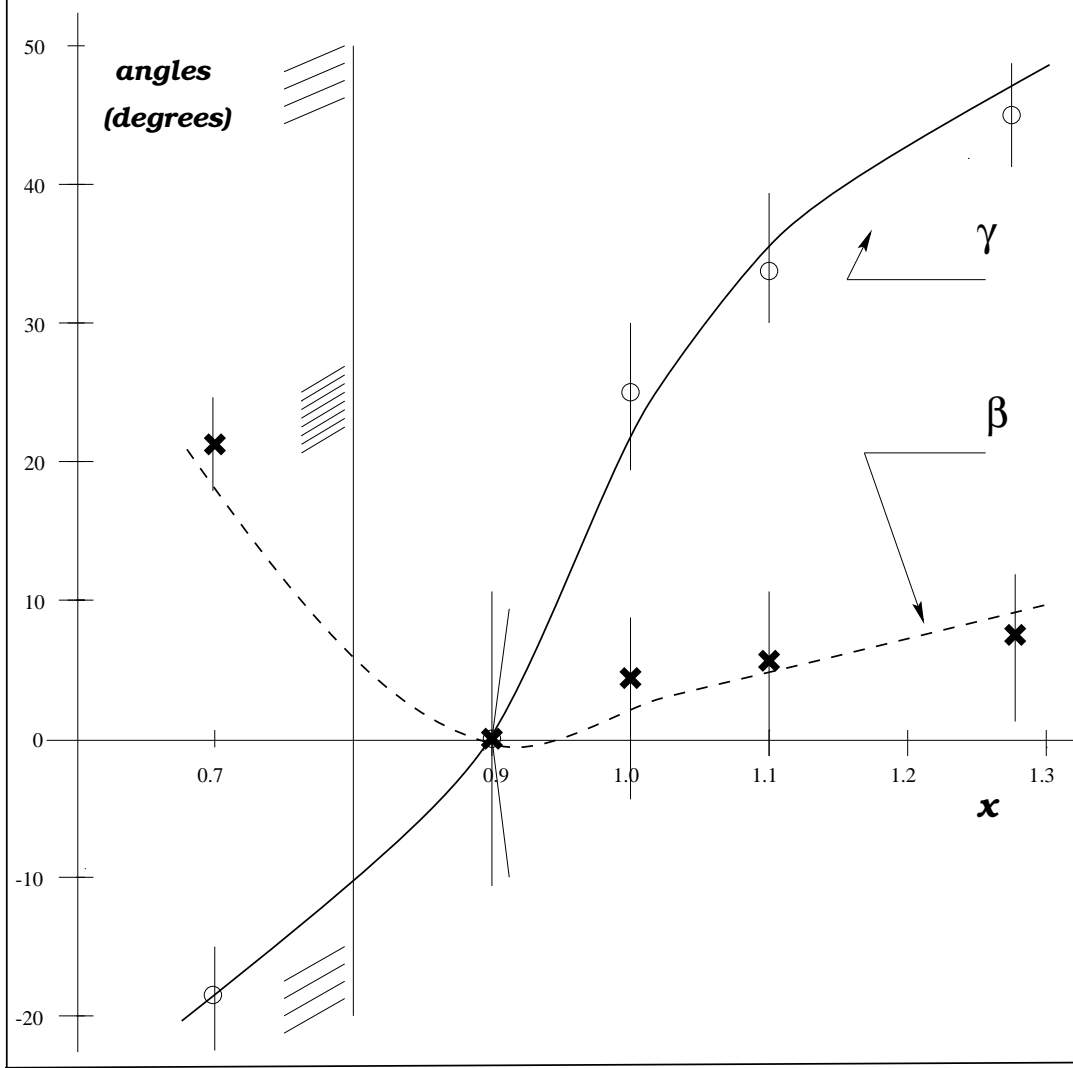


Figure 2: The angles providing the coupling of  $\eta$  and  $\eta'$  to glue (actually, to any singlet state not constituted of  $u$ ,  $d$  and  $s$  quarks) as a function of the symmetry breaking parameter  $x$ .  $x = 1$  corresponds to exact nonet symmetry for the pseudoscalar mesons couplings. A non-zero  $\beta$  is tightly connected with glue in the  $\eta$  meson, while a non-zero  $\gamma$  is tightly connected with glue in the  $\eta'$  meson.

measurements obtained in  $\gamma\gamma$  experiments, while the PDG information reported for  $\eta \rightarrow \gamma\gamma$  branching fraction is the official one [19], somehow influenced by the Primakoff measurement.

The single clear disagreement of model predictions with data concerns the branching fraction for  $K^{*\pm} \rightarrow K^\pm \gamma$ , that we find about half of the reported value in the Review of Particle Properties [19]. We postpone to Section 7 the reexamination of this question.

The recent measurements for  $\phi \rightarrow \eta' \gamma$  are also well accepted by the fit. However, the prediction tends to indicate that the central value found by SND Collaboration [21] is favored compared to that of the CMD2 Collaboration [20].

All this leads us to conclude that the model of symmetry breaking we have presented provides a consistent description of the data. At their present level of accuracy, these do not seem to require additional symmetry breaking effects. An especially satisfactory conclusion is that SU(3) breaking effects are not left free in the fits and are practically determined by the ratio  $f_K/f_\pi$ . Some nonet symmetry breaking and/or glue is needed (see however Ref. [18]).

## 7 The $K^{*\pm}$ Radiative Decay Problem

As shown by the two leftmost data columns in Table 3, the two variants of the model presented above do not account for the accepted [19] radiative decay width  $K^{*\pm} \rightarrow K^\pm \gamma$ . One cannot exclude that this measurement might have to be improved by other means than the Primakoff effect. However, this decay mode has been measured separately for the two charged modes and found to agree with each other. Therefore, the possibility that this failure indicates that models have to be refined cannot be avoided.

The first point which comes to mind is whether the disagreement reported above (a factor of two between prediction and measurement) could be attributed to (missing) SU(2) flavor symmetry breaking effects. If one takes into account the quark content of the  $K^*$ 's, the answer is seemingly no. Indeed, in this case, one could guess that significant unaccounted for SU(2) breaking effects would have rather affected the quality of predictions for  $K^{*0}$  rather than for  $K^{*\pm}$ .

This possibility seeming unlikely, the question becomes : can the VMD modelling we developed be modified in order to account for this mode within an extended SU(3) breaking framework? The reply is positive and is the following.

### 7.1 The $K^*$ Model

Within the spirit of the BKY mechanism, the (unbroken) FKTUY Lagrangian given in Rel. (2) can be broken straightforwardly in three different ways. The first mean is the pseudoscalar field renormalization (see Rel. (7)), which leads to introduce the matrix  $X_A$  and thus the breaking parameter  $\ell_A$  found equal to  $(f_K/f_\pi)^2$  as expected [9]. It has been successfully supplemented with nonet symmetry breaking (and/or glue).

A second mean has been proposed by Ref. [5] (referred to as BGP breaking). It turns

to introduce a breaking matrix  $X_W = \text{diag}(1, 1, 1 + c_W)$  and a new breaking parameter  $\ell_W = 1 + c_W$  in a symmetric way inside the FKTUY Lagrangian<sup>5</sup> :

$$\mathcal{L} = -\frac{3g^2}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} \text{Tr}[\partial_\mu V_\nu X_W \partial_\rho V_\sigma X_A^{-1/2} P' X_A^{-1/2}] \quad (15)$$

In Ref. [17], it has been shown that, supplementing the BKY breaking  $X_A$ , the BGP breaking  $X_W$  alone is unable to account for the  $K^{*\pm}$  radiative decays. Moreover, the constant  $\ell_W$  is pushed to 1 by the fit procedure, and then to no BGP breaking ( $c_W = 0$ ).

A third mean is however conceivable. One should note that the BKY breaking mechanism [9] implies a renormalization (or redefinition) of the pseudoscalar field matrix expressed through  $X_A$  ; however, the  $X_W$  breaking [9, 10] does not end up with a renormalization of the vector field matrix, which remains unchanged in the breaking procedure. One can then *postulate* that the vector meson field matrix has also to be SU(3) broken and also in a symmetric way. This is done by performing the change :

$$V \longrightarrow X_T V X_T \quad , \quad [X_T = \text{diag}(1, 1, 1 + c_T)] \quad (16)$$

in Rel. (15), in complete analogy with the renormalization of the  $P$  matrix. A lack of fancy (not still a mathematical proof) seems to indicate that no fourth mechanism can play.

A detailed study of the consequences of Lagrangian (16) has been performed in Ref. [17] with an interesting conclusion : if one fixes  $\ell_A = 1.5$  as expected [9, 10], the  $X_T$  and  $X_W$  breaking are so sharply correlated that they cannot be left free together. More interestingly, it was found phenomenologically that the additional breaking parameters fulfill :

$$(1 + c_W)(1 + c_T)^4 = 1 \quad (17)$$

This tells us that the most general form of the broken FKTUY Lagrangian accepted by the data can be symbolically written :

$$\mathcal{L} = C \text{Tr}[(X_T^{-1} V X_T)(X_A^{-1/2} P' X_A^{-1/2})(X_T V X_T^{-1})] \quad (18)$$

In this case, all couplings constants write as when having solely the BKY breaking mechanism, supplemented with nonet symmetry breaking and/or glue, except for the  $K^*$  decay modes which become :

$$\begin{cases} G_{K^{*0}K^0\gamma} = & - G \frac{\sqrt{K'}}{3} (1 + \frac{1}{\ell_T}) \\ G_{K^{*\pm}K^\pm\gamma} = & G \frac{\sqrt{K'}}{3} (2 - \frac{1}{\ell_T}) \end{cases} \quad (19)$$

---

<sup>5</sup>The symmetry can be made manifest. What is written in Rel. (15), can be symbolically written  $\text{Tr}[V X_W V (X_A^{-1/2} P X_A^{-1/2})]$  and is obviously identical to  $\text{Tr}[X_W^{1/2} V (X_A^{-1/2} P X_A^{-1/2}) V X_W^{1/2}]$ .

where  $K' = \ell_T/\ell_A$  and  $\ell_T = (1+c_T)^2$ . Stated otherwise, both  $K^*$  couplings are changed : the one correctly accounted for by the previous modellings ( $K^{*0}$ ), *and* the one poorly described ( $K^{*\pm}$ ). Therefore, a fit value for  $\ell_T$  must change  $G_{K^{*\pm}}$  while leaving  $G_{K^{*0}}$  practically unchanged, despite the *functional* relation among them.

Assuming no coupling to glue, we have performed the fit and found a perfect fit quality ( $\chi^2/dof = 11.07/10$ ) with practically the same parameter values as in the models above and additionally :

$$\ell_T = 1.19 \pm 0.06 \quad , \quad (c_T = 0.109 \pm 0.024) \quad (20)$$

The predicted branching fractions are given in the third data column in Table 3. They indeed show that all predictions (including for the  $K^{*0}$  mode) are unaffected except for the  $K^{*\pm}$  mode, now in quite nice agreement with its accepted value [19].

One may wonder that the  $K^{*0}$  mode is unchanged, while the  $K^{*\pm}$  mode is increased by a factor  $\simeq 2$ . For this purpose, one may compare the values of the  $\ell_T$  part of the couplings in Rels. (19) at  $\ell_T = 1$  and at  $\ell_T = 1.2$ . One thus find that the former change is  $2 \rightarrow 2.01$ , while the later change is  $1 \rightarrow 1.28$ . Therefore, the change requested in order to account for the  $K^{*\pm}$  mode, results in an insignificant change for the  $K^{*0}$  mode.

Therefore, quite unexpectedly, a tiny change in the VMD model we have shown is enough to describe indeed all radiative decays at their presently accepted values.

Nevertheless, the additional mechanism complicates the full breaking picture which is otherwise quite simple. One can hope that new measurements for the  $K^{*\pm}$  radiative decay will come soon and tell definitely whether this complication really proceeds from physics.

## 7.2 The $K^*$ Model and the WZW Lagrangian

In Section 3.4, we have remarked that imposing the change of fields from  $P$  to  $P'$  to the WZW Lagrangian (see Rel. (11)) provides the same description of radiative decays of pseudoscalar mesons than the broken HLS–FKTUY model. Thus, one has checked that that these two descriptions were indeed equivalent. The VMD description is however able to connect the  $P\gamma\gamma$  couplings to the  $VP\gamma$  ones with the success illustrated by Table 2.

When introducing the additional breaking schemes in order to construct the  $K^*$  model sketched above (the expanded Lagrangian can be found in the Appendix of Ref. [17]), this property is formally lost, except if additionally to the change  $P \rightarrow P'$ , we also perform the change  $Q^2 \rightarrow X_W X_T^4 Q^2$ , e.g. if we “renormalize” the SU(3) charge matrix, or the WZW Lagrangian as a whole. It happens that the condition in Rel. (17) prevents such an ugly transform. Stated otherwise, phenomenology forces a relation which is such that the two-photon decays of pseudoscalar mesons are still given by the Wess–Zumino–Witten Lagrangian [24,25], with breaking only for the single occuring matter field matrix  $P$ .

In order that the equivalence between VMD and WZW is generally maintained, the  $K^*$  breaking should affect the  $K^*$  couplings only. If, instead, the other couplings  $VP\gamma$  were affected, this would propagate down to the  $P\gamma\gamma$  couplings. In this case, the equivalence statement between (broken) VMD and (broken) WZW would no longer hold. Thus,



the  $K^*$  breaking scheme seems indeed the most general consistent with this equivalence statement.

## 8 Conclusion

The VMD based model we have presented indeed describes all radiative decays  $VP\gamma$  and  $P\gamma\gamma$  accurately. This model relies heavily on the HLS model supplemented with the BKY breaking mechanism in order to account for  $SU(3)$  symmetry breaking. It should be stressed that phenomenology indeed confirms the theoretical connection between the  $SU(3)$  breaking parameter and the ratio  $f_K/f_\pi$ .

One has additionally to introduce a further degree of freedom ; this can be either of direct nonet symmetry breaking ( $x \neq 1$ ) or coupling of the  $\eta'$  meson to an additional singlet ( $\gamma \neq 0$ ). A mixture of both effects is also an acceptable solution, numerically and theoretically. Actually, radiative decays of light mesons alone cannot provide more detailed information about this possible additional singlet without additional theoretical input.

The picture that emerges from there is quite consistent and tends to indicate that present data do not require any breaking of the  $SU(2)$  symmetry at a visible level in only radiative decays of light mesons.

The single present datum which requires special additional input is the  $K^{*\pm}$  radiative decay. It can be done succesfully without destroying an equivalence statement between VMD and the WZW description of pseudoscalar meson decays to two photons. However, a confirmation of the present data for the  $K^{*\pm}$  radiative decay looks desirable.

Anyway, whatever is the precise value of the  $K^{*\pm}$  radiative width, VMD expressed through the general concept underlying the HLS model is able to provide a quite consistent picture of the radiative decays of all light mesons.

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Process	Nonet Sym. + SU(3)	Glue + SU(3)	$K^{*\pm}$ Breaking	PDG
$\rho \rightarrow \pi^0 \gamma (\times 10^4)$	$5.16 \pm 0.03$	$5.16 \pm 0.03$	$5.16 \pm 0.03$	$6.8 \pm 1.7$
$\rho \rightarrow \pi^\pm \gamma (\times 10^4)$	$5.12 \pm 0.03$	$5.12 \pm 0.03$	$5.12 \pm 0.03$	$4.5 \pm 0.5$
$\rho \rightarrow \eta \gamma (\times 10^4)$	$3.25 \pm 0.10$	$3.28 \pm 0.10$	$3.31 \pm 0.09$	$2.4^{+0.8}_{-0.9}$
$\eta' \rightarrow \rho \gamma (\times 10^2)$	$33.1 \pm 2.0$	$33.7 \pm 2.0$	$33.0 \pm 1.8$	$30.2 \pm 1.3$
$K^{*\pm} \rightarrow K^\pm \gamma (\times 10^4)$	$5.66 \pm 0.03$	$5.66 \pm 0.03$	$9.80 \pm 0.93$	$9.9 \pm 0.9$
$K^{*0} \rightarrow K^0 \gamma (\times 10^3)$	$2.30 \pm 0.01$	$2.30 \pm 0.01$	$2.32 \pm 0.02$	$2.3 \pm 0.2$
$\omega \rightarrow \pi^0 \gamma (\times 10^2)$	$8.50 \pm 0.05$	$8.50 \pm 0.05$	$8.50 \pm 0.05$	$8.5 \pm 0.5$
$\omega \rightarrow \eta \gamma (\times 10^4)$	$8.0 \pm 0.2$	$8.1 \pm 0.2$	$8.12 \pm 0.19$	$6.5 \pm 1.0$
$\eta' \rightarrow \omega \gamma (\times 10^2)$	$2.8 \pm 0.2$	$2.9 \pm 0.2$	$2.8 \pm 0.2$	$3.01 \pm 0.30$
$\phi \rightarrow \pi^0 \gamma (\times 10^3)$	$1.27 \pm 0.13$	$1.28 \pm 0.12$	$1.26 \pm 0.13$	$1.31 \pm 0.13$
$\phi \rightarrow \eta \gamma (\times 10^2)$	$1.25 \pm 0.04$	$1.25 \pm 0.05$	$1.22 \pm 0.04$	$1.26 \pm 0.06$
$\phi \rightarrow \eta' \gamma (\times 10^4)$	$0.61 \pm 0.027$	$0.55 \pm 0.03$	$0.63 \pm 0.02$	$1.2^{+0.7}_{-0.5}$
$\eta \rightarrow \gamma \gamma (\times 10^2)$	$40.5 \pm 1.7$	$40.8 \pm 1.8$	$41.5 \pm 1.4$	$39.21 \pm 0.34$
$\eta' \rightarrow \gamma \gamma (\times 10^2)$	$2.1 \pm 0.1$	$2.1 \pm 0.1$	$2.1 \pm 0.1$	$2.11 \pm 0.13$

**Table 3** : Branching fractions from fits for radiative decays under various conditions of symmetry breakings. Note that the rate for  $K^{*\pm}$  is a prediction in the first two data columns, while the corresponding data is included in the fit which leads to the third data column.

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